FIITJEE
ALL INDIA TEST SERIES
JEE (Advanced)-2024
OPEN TEST – II
PAPER –1
TEST DATE: 21-04-2024

ANSWERS, HINTS & SOLUTIONS

Physics

PART – I

SECTION – A

1. A, C

Sol. The induced electric field in the cylinder,

$$E = -\frac{r}{2}\frac{dB}{dt} \quad \ldots(i)$$

The induced current in the cylinder,

$I = j\ell d$

$$I = \frac{E}{\rho}\ell d$$

$$I = -\left(\frac{r}{2}\frac{dB}{dt}\right)\frac{\ell d}{\rho}$$

$$\frac{B\ell}{\mu_0} = -\left(\frac{r}{2}\frac{dB}{dt}\right)\frac{\ell d}{\rho} \qquad \left(\because\ B = \frac{\mu_0 I}{\ell}\right)$$

$$\int_{B_0}^{B}\frac{dB}{B} = -\frac{2\rho}{\mu_0 rd}\int_0^t dt$$

$$\ell n\left(\frac{B}{B_0}\right) = -\frac{2\rho t}{\mu_0 rd}$$

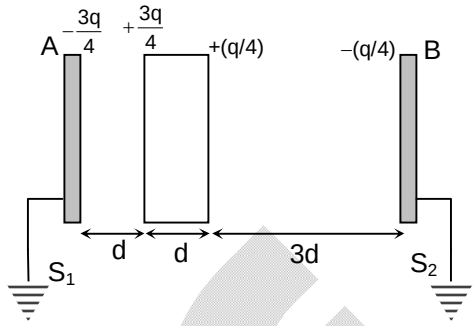
$$B = B_0 e^{-\frac{2\rho t}{\mu_0 rd}}$$

The induced current in the cylinder,

$$I = \frac{B\ell}{\mu_0} \Rightarrow I = \left(\frac{B_0\ell}{\mu_0}\right)e^{-\frac{2\rho t}{\mu_0 rd}}$$

2. B, D

Sol. After both the switches S_1 and S_2 are simultaneoulsy closed.



$C_1 = \frac{\varepsilon_0 A}{d}$

$C_2 = \frac{\varepsilon_0 A}{3d}$

$C_1 : C_2 = 3 : 1$

Hence, $q_1 = \frac{3q}{4}$ and $q_2 = \frac{q}{4}$

The charge flown through the switch S_1, $\Delta Q_1 = 2q - \left(-\frac{3q}{4}\right) = \frac{11q}{4}$ (from the plate A to the earth)

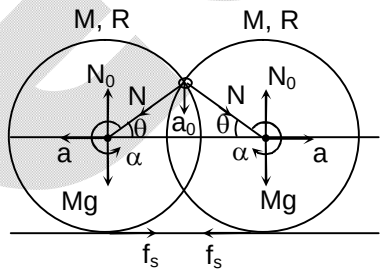
The charge flown through the switch S_2, $\Delta Q_2 = -\frac{q}{4} - (-q) = \frac{3q}{4}$ (from the earth to the plate B)

3. A, B, D

Sol. $v \cos\theta = v_0 \sin\theta$

$v = v_0 \tan\theta$ …(i)

$a = a_0 \tan\theta + v_0 \sec^2\theta\left(\frac{d\theta}{dt}\right)$



At $t = 0$, $v_0 = 0$ and $\theta = 37°$

$a = a_0 \tan\theta$

$a_0 = \frac{4a}{3}$ …(ii)

$N\cos\theta - f_s = Ma$ …(iii)

$f_S R = MR^2\alpha \Rightarrow f_S = Ma$ …(iv) ($\because$ $a = \alpha R$)

Adding equations (iii) and (iv), we get

$N\cos\theta = 2Ma$

$\Rightarrow \frac{4N}{5} = 2 \times 4a$

$\Rightarrow N = 10\,a$ …(v)

Now,

$mg - 2N\sin\theta = ma_0$

$1 \times 10 - 2 \times 10a \times \frac{3}{5} = 1 \times \frac{4a}{3}$ {from equation (ii) and (v)}

$\Rightarrow 10 = 12a + \frac{4a}{3}$

$\Rightarrow 10 = \frac{40a}{3} \Rightarrow a = \frac{3}{4}$ m/s^2 and $a_0 = \frac{4a}{3} = \frac{4}{3} \times \frac{3}{4} = 1 \Rightarrow a_0 = 1$ m/s^2

Now, $f_S = Ma = 4 \times \frac{3}{4} = 3$ N

4. C

Sol. The maximum speed will be acquired by each block in equilibrium.

Let the spring is stretched by x

When the system is in equilibrium,

$T = 2mg$ …(i)

$T = kx + \mu mg$ …(ii)

Solving equation (i) and (ii), we get

$2mg = kx + \mu mg$

$$kx = 2mg - \frac{mg}{2} \Rightarrow x = \frac{3mg}{2k}$$

Now using work energy theorem for the system

$$2mgx - \mu mgx = \frac{1}{2}kx^2 + \frac{1}{2}(m+2m)v^2$$

$$(2mg - \mu mg)x = \frac{kx^2}{2} + \frac{1}{2} \times 3mv^2$$

$$kx^2 = \frac{kx^2}{2} + \frac{3mv^2}{2}$$

$$\frac{kx^2}{2} = \frac{3mv^2}{2}$$

$$v = x\sqrt{\frac{k}{3m}} = \frac{3mg}{2k}\sqrt{\frac{k}{3m}}$$

$$v = \frac{g}{2}\sqrt{\frac{3m}{k}}$$

5. A

Sol. $$U = \frac{a}{x^4} - \frac{b}{x^2}$$

$$F_x = -\frac{dU}{dx} = \frac{4a}{x^5} - \frac{2b}{x^3}$$ …(i)

At equilibrium, $F_x = 0$

$$\frac{4a}{x^5} - \frac{2b}{x^3} = 0$$

$$x_0 = \sqrt{\frac{2a}{b}}$$ …(ii)

Now, $$F_x = \frac{4a}{x^5} - \frac{2b}{x^3}$$

$$\frac{dF_x}{dx} = -\frac{20a}{x^6} + \frac{6b}{x^4}$$

At $x = x_0 = \sqrt{\frac{2a}{b}}$

$$\frac{dF_x}{dx} = -\frac{20ab^3}{8a^3} + \frac{6b \times b^2}{4a^2}$$

$$\frac{dF_x}{dx} = -\left(\frac{b^3}{a^2}\right)$$

$$dF_x = -\left(\frac{b^3}{a^2}\right)dx$$

$$m\frac{d^2r}{dt^2} = -\left(\frac{b^3}{a^2}\right)r \qquad \text{(where } r = x - x_0 \text{ and } r << x_0\text{)}$$

$$\frac{d^2r}{dt^2} = -\left(\frac{b^3}{ma^2}\right)r$$

Time period, $T = 2\pi\sqrt{\frac{ma^2}{b^3}}$

6. C

Sol. $\frac{Ed}{3} + 0 + \frac{Ed}{3} = V$

$$\frac{2Ed}{3} = V \Rightarrow E = \frac{3V}{2d} \qquad \ldots(i)$$

$$\frac{q}{K\in_0 A} = \frac{3V}{2d}$$

$$q = K\in_0 A\left(\frac{3V}{2d}\right) \qquad \ldots(ii)$$

The modulus of the force of electrostatic interaction on each plate,

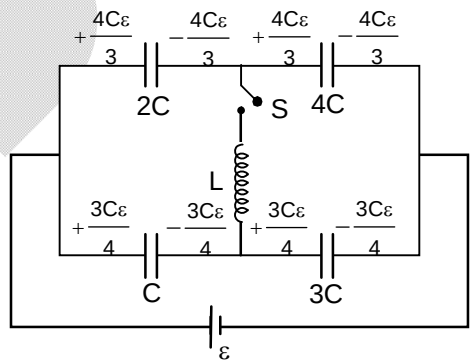
$$F = \frac{q^2}{2\in_0 A} = \frac{9\in_0 AK^2V^2}{8d^2}$$

7. B

Sol. Before the swtich 'S' is closed.

$$C_{eq} = \frac{4C}{3} + \frac{3C}{4} = \frac{25C}{12}$$

$$Q = C_{eq}\varepsilon = \frac{25C\varepsilon}{12}$$



When the current through the inductor is maximum after the switch 'S' is closed.

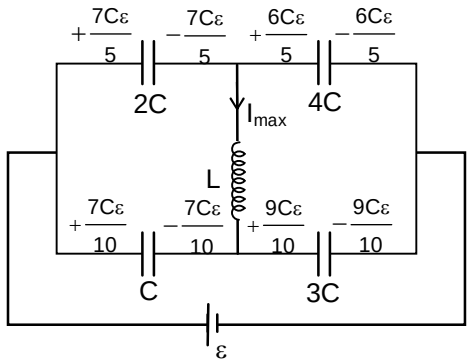
$$C'_{eq} = \frac{3C \times 7C}{10C} = \frac{21C}{10}$$

$$Q' = C'_{eq} = \frac{21C\varepsilon}{10}$$

Charge flown through the battery after the swtich 'S' is closed.

$$\Delta Q = \left(\frac{21C\varepsilon}{10}\right) - \left(\frac{25C\varepsilon}{12}\right)$$

$$\Delta Q = \frac{C\varepsilon}{60}$$



Now, $\Delta W_b = \Delta U + \frac{1}{2}LI_{max}^2$

$$\frac{C\varepsilon^2}{60} = \frac{1}{2}\left(\frac{21C}{10} - \frac{25C}{12}\right)\varepsilon^2 + \frac{1}{2}LI_{max}^2$$

$$\frac{C\varepsilon^2}{60} = \frac{C\varepsilon^2}{120} + \frac{1}{2}LI_{max}^2$$

$$\Rightarrow \frac{C\varepsilon^2}{120} = \frac{1}{2}LI_{max}^2$$

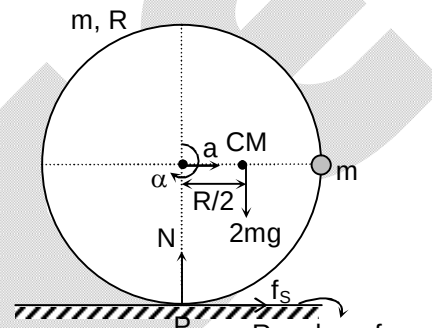
$$\Rightarrow I_{max} = \frac{\varepsilon}{2}\sqrt{\frac{C}{15L}}$$

8. A

Sol. $I_P = 2mR^2 + 2mR^2 = 4mR^2$

Using conservation of energy

$$mg\frac{R}{2} = \frac{1}{2} \times 4mR^2\omega^2 \Rightarrow \omega = \sqrt{\frac{g}{4R}} \quad \ldots(i)$$

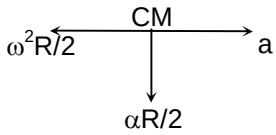


Now, $\tau_P = I_P\alpha \Rightarrow 2m(g + \omega^2 R)\frac{R}{2} = 4mR^2\alpha$

$$\Rightarrow \left(g + \frac{g}{4}\right) = 4R\alpha \Rightarrow \alpha = \frac{5g}{16R} \quad \ldots(ii)$$

$$2mg - N = 2m\frac{\alpha R}{2} \Rightarrow 2mg - N = \frac{5mg}{16}$$

$a = \alpha R$



$$N = \frac{27mg}{16} = 13.5\ N$$

$$f_s = 2m\left(a - \frac{\omega^2 R}{2}\right) = 2m\left(\frac{5g}{16} - \frac{g}{8}\right) = \frac{3mg}{8} = 3N$$

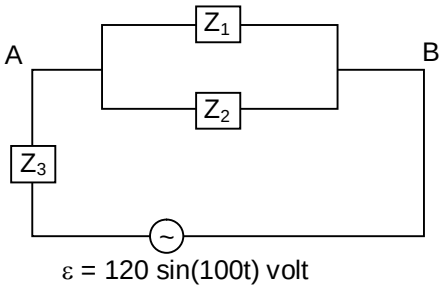
$$F_h = m(\alpha R - \omega^2 R) = m\left(\frac{5g}{16} - \frac{g}{4}\right) = \frac{mg}{16} = 0.5\ N$$

$$mg - F_v = m\alpha R \Rightarrow F_v = mg - \frac{5mg}{16} = \frac{11mg}{16} = 5.5\ N$$

9. C

Sol. $Z_1 = R_1 + \omega Li = 20 + 20i$

$$Z_2 = R_2 - \frac{1}{\omega C}i = 20 - \frac{1}{100 \times 5 \times 10^{-4}}i = 20 - 20i$$



$$\frac{1}{Z'} = \frac{1}{Z_1} + \frac{1}{Z_2}$$

$$\frac{1}{Z'} = \frac{1}{20(1+i)} + \frac{1}{20(1-i)}$$

$$\frac{1}{Z'} = \frac{(1-i)}{40} + \frac{(1+i)}{40} \Rightarrow \frac{1}{Z'} = \frac{2}{40}$$

$\Rightarrow Z' = 20\ \Omega$

Hence $Z = Z' + Z_3 = 20 + (10 + 30i)$

$\Rightarrow Z = 30(1+i)$

The peak current through the source, $I_0 = \frac{\varepsilon_0}{|Z|} = \frac{120}{30\sqrt{2}} = 2\sqrt{2}$ Amp

The instantaneous current through the source, $i = 2\sqrt{2}\sin\left(100t - \frac{\pi}{4}\right)$

The instantaneous potential drop between A and B, $v' = 40\sqrt{2}\sin\left(100t - \frac{\pi}{4}\right)$

The peak current through the resistor R_2, $I_2 = \frac{40\sqrt{2}}{20\sqrt{2}} = 2$ amp

The average power dissipated in the resistor R_3, $P_3 = I_{rms}^2 R_3$

$\Rightarrow \quad P_3 = \left(\frac{I_0}{\sqrt{2}}\right)^2 R_3 = (2)^2 \times 10 = 40$ watt

10. B

Sol. $v_1\cos 30° + v_2\cos 60° = v$

$v_1\frac{\sqrt{3}}{2} + \frac{v_2}{2} = v$

$\sqrt{3}v_1 + v_2 = 2v$ …(i)

Using conservation of momentum along x and y axes

$mu = m(v\cos 60° + u\cos^2 30°) + 2mv_2$

$u = \frac{v}{2} + \frac{3u}{4} + 2v_2 \Rightarrow 8v_2 + 2v = u$ …(ii)

Also, $mv_1 + m(v\cos 30° - u\cos 30°\cos 60°) = 0$

$v_1 + \frac{v\sqrt{3}}{2} = \frac{\sqrt{3}}{4}u \Rightarrow 4v_1 + 2\sqrt{3}v = \sqrt{3}u$ …(iii)

Solving equations (ii) and (iii), we get

$4v_1 + \sqrt{3}(u - 8v_2) = \sqrt{3}u$

$4v_1 = 8\sqrt{3}v_2$

$\Rightarrow v_1 = 2\sqrt{3}v_2$

Solving equation (i) and (ii), we get

$\sqrt{3}v_1 + v_2 = u - 8v_2$

$\Rightarrow 6v_2 + 9v_2 = u$

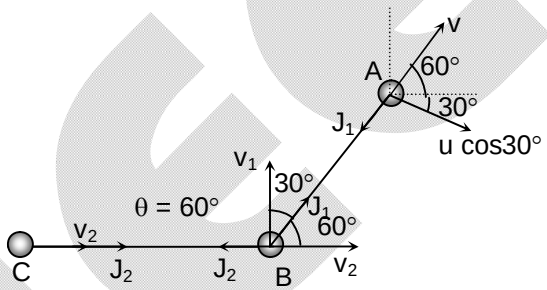
$\Rightarrow v_2 = \frac{u}{15} = \frac{30}{15} = 2$ m/s and $v_1 = 2\sqrt{3}v_2 = 4\sqrt{3}$ m/s

Hence, $v_B = \sqrt{v_1^2 + v_2^2} = \sqrt{48 + 4} = 2\sqrt{13} = 7.2$ m/s

$v_C = v_2 = 2$ m/s

Now, $J_1\cos 60° = 2mv_2$ Å $J_1 = 4mv_2 = 16$ N-s

$J_2 = mv_2 = 2 \times 2 = 4$ N-s



11. D

Sol. $E_1 = \frac{\lambda_1}{2\pi\varepsilon_0 r_1}$ and $E_2 = \frac{\lambda_2}{2\pi\varepsilon_0 r_2}$

$F_1 = P\left|\frac{dE_1}{dr_1}\right| = P\left|\frac{\lambda_1}{2\pi\varepsilon_0 r_1^2}\right|$ and $F_2 = P\left|\frac{dE_2}{dr_2}\right| = P\left|\frac{\lambda_2}{2\pi\varepsilon_0 r_2^2}\right|$

(P) $F_1 = \frac{P2\lambda \times 4}{2\pi\varepsilon_0 d^2} = \frac{4\lambda P}{\pi\varepsilon_0 d^2}$ and $F_2 = \frac{P\lambda 4}{2\pi\varepsilon_0 d^2} = \frac{2\lambda P}{\pi\varepsilon_0 d^2}$

Hence, $F = F_1 + F_2 = \dfrac{6\lambda P}{\pi\varepsilon_0 d^2}$

(Q) $F_1 = \dfrac{4\lambda P}{\pi\varepsilon_0 d^2}$ and $F_2 = \dfrac{2\lambda P}{\pi\varepsilon_0 d^2}$

Hence, $F = F_1 - F_2 = \dfrac{2\lambda P}{\pi\varepsilon_0 d^2}$

(R) $F_1 = \dfrac{P\lambda \times 16}{2\pi\varepsilon_0 d^2} = \dfrac{8\lambda P}{\pi\varepsilon_0 d^2}$ and $F_2 = \dfrac{P2\lambda \times 9}{2\pi\varepsilon_0 d^2} = \dfrac{9\lambda P}{\pi\varepsilon_0 d^2}$

Hence, $F = F_1 + F_2 = \dfrac{17\lambda P}{\pi\varepsilon_0 d^2}$

(S) $F_1 = \dfrac{8P\lambda}{\pi\varepsilon_0 d^2}$ and $F_2 = \dfrac{9P\lambda}{\pi\varepsilon_0 d^2}$

Hence, $F = F_2 - F_1 = \dfrac{\lambda P}{\pi\varepsilon_0 d^2}$

SECTION – B

12. 21

Sol. The torque due to the viscous liquid on the top of the solid cylinder

$$\tau_1 = \int_0^R \eta 2\pi r dr \frac{\omega r}{h} r$$

$$\tau_1 = 2\pi\eta \frac{\omega}{h} \int_0^R r^3 dr$$

$$\tau_1 = 2\pi\eta \frac{\omega}{h} \frac{R^4}{4} = \frac{\pi\eta\omega R^4}{2h} \quad \ldots(i)$$

The torque due to the viscous liquid on the curved surface of the solid cylinder

$$\tau_2 = \eta 2\pi R\ell \frac{\omega R}{h} R$$

$$\tau_2 = 2\pi\eta R \times 5R \frac{\omega R^2}{h}$$

$$\tau_2 = 10\pi\eta \frac{\omega R^4}{h} \quad \ldots(ii)$$

Hence the net torque required,

$\tau = \tau_1 + \tau_2$

$$\tau = \left(\frac{\pi\eta\omega R^4}{2h}\right) + \left(\frac{10\pi\eta\omega R^4}{h}\right)$$

$$\tau = \frac{21\pi\eta\omega R^4}{2h}$$

Hence, K = 21

13. 12

Sol. $\dfrac{1}{f_{\ell_1}} = (1.6 - 1)\left(\dfrac{1}{20} - \dfrac{1}{30}\right)$

$$\frac{1}{f_{\ell_2}} = 0.6 \times \frac{1}{60} \Rightarrow f_{\ell_1} = 100 \text{ cm}$$

$$\frac{1}{f_{\ell_2}} = \left(\frac{4}{3} - 1\right)\left(\frac{1}{30} + \frac{1}{60}\right)$$

$$\frac{1}{f_{\ell_2}} = \frac{1}{3} \times \frac{3}{60} \Rightarrow f_{\ell_2} = 60 \text{ cm}$$

$$\frac{1}{f_{\ell_3}} = (1.8 - 1)\left(-\frac{1}{60} + \frac{1}{40}\right)$$

$$\frac{1}{f_{\ell_3}} = 0.8 \times \frac{1}{120} \Rightarrow f_{\ell_3} = 150 \text{ cm}$$

Now, $-\frac{1}{F} = \frac{2}{f_{\ell_1}} + \frac{2}{f\ell_2} + \frac{2}{f\ell_3} - \frac{1}{f_m}$

$$-\frac{1}{F} = \frac{2}{100} + \frac{2}{60} + \frac{2}{150} + \frac{1}{20}$$

$$\Rightarrow F = -\frac{60}{7} \text{ cm}$$

Now using mirror formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{F}$$

$$\Rightarrow \frac{1}{v} - \frac{1}{30} = \frac{-7}{60}$$

$$\Rightarrow v = -12 \text{ cm}$$

14. 420

Sol. the frequency of sound received by the reflector car B,

$$f_1 = \left(\frac{v + \omega + u}{v + \omega - u}\right) f \qquad \ldots(i)$$

The frequency of the reflected sound wave received by the driver of car A

$$f_2 = \left(\frac{v - \omega + u}{v - \omega - u}\right) f_1$$

$$f_2 = \left(\frac{v - \omega + u}{v - \omega - u}\right)\left(\frac{v + \omega + u}{v + \omega - u}\right) f \qquad \ldots(ii)$$

$$f_2 = \left(\frac{330 - 5 + 25}{330 - 5 - 25}\right)\left(\frac{330 + 5 + 25}{330 + 5 - 25}\right) 310 \qquad \ldots(ii)$$

$$f_2 = \frac{350}{300} \times \frac{360}{310} \times 310$$

$f_2 = 420$ Hz

15. 54

Sol. Time period of Earth's revolution around the Sun

$$T = \frac{2\pi}{\sqrt{GM_S}} R^{3/2} \qquad \ldots(i)$$

Time period of the double stars binary system

$$T_b = \frac{2\pi \times (36R)^{3/2}}{\sqrt{G(4M_S + 12M_S)}}$$

$$T_b = \frac{2\pi \times (6)^3 R^{3/2}}{\sqrt{G16M_S}}$$

$$T_b = \frac{216}{4} \times \frac{2\pi R^{3/2}}{\sqrt{GM_S}}$$

$T_b = 54T$

$n = 54$

16. 55

Sol. ${}_1^2H + {}_1^2H \longrightarrow {}_1^3H + {}_1^1P$

The Q-value of the reaction

$Q = (2.8 \times 3) - (1 \times 4)$

$\Rightarrow Q = 4.4$ MeV

Using conservation of momentum

$\vec{P}_1 + \vec{P}_2 = 0 \Rightarrow \vec{P}_1 = -\vec{P}_2$

$P_1^2 = P_2^2$

$2m_1K_1 = 2m_2K_2$

$K_1 = 3K_2$ …(i)

Also, $K_1 + K_2 = Q$

$K_1 + K_2 = 4.4$ …(ii)

$4K_2 = 4.4 \Rightarrow K_2 = 1.1$ MeV and $K_1 = 3.3$ MeV

Hence $(10\ K_1 + 20\ K_2) = (10 \times 3.3) + (20 \times 1.1) = (33+22) = 55$ MeV

17. 7

Sol. $C_{eq} = \frac{C_1C_2}{C_1 + C_2} = \frac{10 \times 40}{10 + 40} = 8\mu F$ …(i)

Now, $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$

$$\frac{dC_{eq}}{C_{eq}^2} = \frac{dC_1}{C_1^2} + \frac{dC_2}{C_2^2}$$

$$\frac{dC_{eq}}{C_{eq}} = \left(\frac{dC_1}{C_1^2} + \frac{dC_2}{C_2^2}\right) C_{eq}$$

$$\frac{dC_{eq}}{C_{eq}} = \left(\frac{0.10}{100} + \frac{0.20}{1600}\right) \times 8 = \frac{1.80}{200} = 9 \times 10^{-3}$$ …(ii)

Now, $U = \frac{1}{2} C_{eq} V^2$

$$\frac{dU}{U} = \frac{dC_{eq}}{C_{eq}} + \frac{2dV}{V} = 9 \times 10^{-3} + 2 \times \frac{0.02}{8} = 14 \times 10^{-3}$$

Hence, $\frac{dU}{U} \times 100 = 14 \times 10^{-3} \times 100 = 1.4\%$

$\Rightarrow n = 1.4$

Hence, $5n = 7$

Chemistry

PART – II

SECTION – A

18. A, C, D

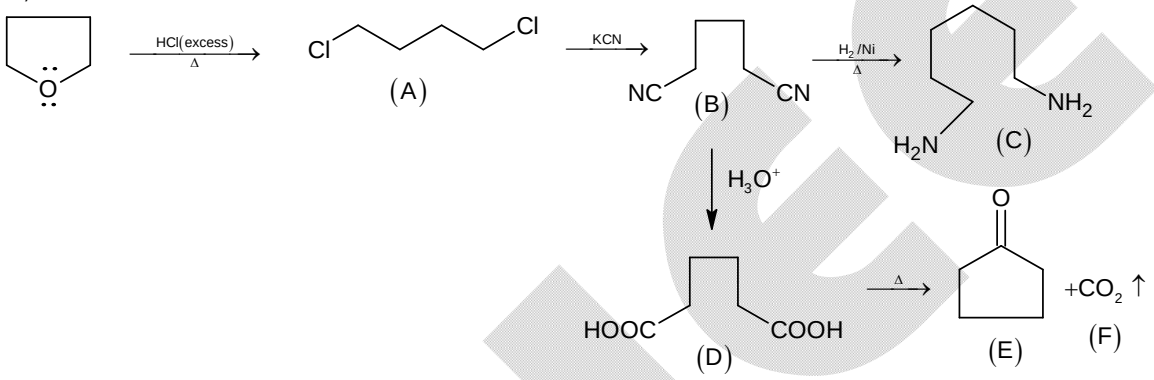
Sol. Free expansion is irreversible adiabatic as well as isothermal process, where

$\Delta G < 0, (\Delta S)_{univ.} > 0.$

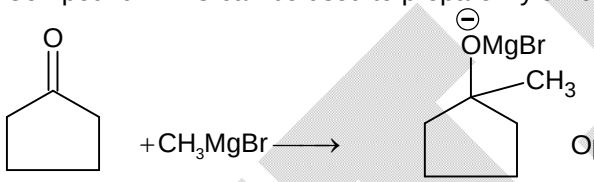
$\Delta T = 0, w = 0, q = 0, \Delta U = 0.\ (\Delta S)_{surr.} = 0, (\Delta S)_{syst.} > 0.$

19. A, C

Sol.



Compound D + C can be used to prepare Nylon-6,6



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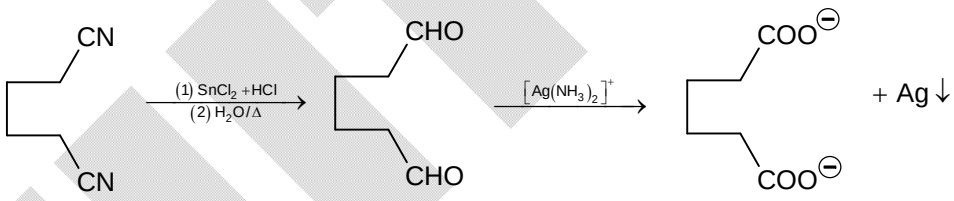


20. A, C, D

Sol. At anode oxidation will takes place and at cathode reduction of NO_3^- in HNO_2 takes place.

Hence, cathodic compartment is acidic.

$$Hg_2^{2+} + NO_3^- + 3H^+ \rightleftharpoons 2Hg^{2+} + HNO_2 + H_2O$$

$$E_{cell} = E^o_{cell} - \frac{0.06}{2}\log\frac{\left[Hg^{2+}\right]^2[HNO_2][H_2O]}{\left[Hg_2^{2+}\right]\left[NO_3^-\right]\left[H^+\right]^3}$$

$$E_{cell} = E^o_{cell} - \frac{0.06}{2}\log\frac{1}{\left[H^+\right]^3}$$ if activity of all the compartment is unity.

$$E_{cell} = 0.02 + \frac{0.06}{2}\log\left[H^+\right]^3$$

$$E_{cell} = 0.02 + 0.03\log\left[H^+\right]^3$$

$0 = 0.02 - 0.09\ pH$

$pH = \left(\frac{2}{9}\right)$

2F charge will deposit 2 mole of Hg^{2+}.
0.6 mole electron will deposit 0.6 mole of Hg^{2+}.

21. B

Sol. $HCl(g) + NH_3(g) \longrightarrow NH_4Cl(s)$

$\frac{2V}{RT}$	$\frac{3V}{2RT}$	0
$\frac{V}{2RT}$	0	0

Final pressure $= \frac{\frac{V}{2RT} \times RT}{2V} = \frac{1}{4}$ atm.

Pressure difference $= 1 - \frac{1}{4} = \frac{3}{4}$ atm.

Difference of Hg level in two column of both Manometers are 57 cm Hg.

22. A

Sol. Integrated rate law $\frac{kt}{2} = \frac{1}{\sqrt{C_t}} - \frac{1}{\sqrt{C_o}}$

$t \to t_{1/2}\ \ C_t = \frac{C_o}{2}$

$t_{1/2} = \frac{2\left(\sqrt{2}-1\right)}{k\sqrt{C_o}}$

23. A

Sol. XeF_5^- is pentagonal planar and non polar molecules.

24. C

Sol. Due to NGP in (II) and in (IV) compound is exo and non-classical carbocation formation due to NGP.

25. A

Sol. (P)

$$Na_3PO_4 + H_3PO_4 \longrightarrow NaH_2PO_4 + Na_2HPO_4$$

mmole	5	5	0	0
	0	0	5	5

$pH = pK_{a_2} + \log\frac{\left[HPO_4^{--}\right]}{\left[H_2PO_4^-\right]} = pK_{a_2} = 8.00$

(Q) Solution will be acidic buffer. After reaction

m_{eq} of $CH_3COOH = 25$

m_{eq} of $CH_3COO^- = 25$

$pH = pK_a + \log\frac{\left[CH_3COO^-\right]}{\left[CH_3COOH\right]} = pK_a = 5.00$

(R) Solution will act as a basic buffer

$$pOH = pK_b + \log\frac{\left[NH_4^+\right]}{\left[NH_4OH\right]} = pK_b + \log\frac{0.02}{0.2} = pK_b - 1 = 4$$

pH = 10.00

(S) It is a case of salt hydrolysis of (strong base + weak acid)

$$pH = 7 + \frac{1}{2}\left(pK_a + \log C\right) = 7 + \frac{1}{2}\left(5 + \log 0.1\right)$$

$$= 7 + \frac{1}{2}\left(5 - 1\right)$$

= 7 + 2

= 9.

26. B

Sol. $XeF_4 + O_2F_2 \longrightarrow XeF_6 + O_2$

$P_4 + 10SO_2Cl_2 \longrightarrow 4PCl_5 + 10SO_2$

$SbF_5 + XeF_4 \longrightarrow \left[XeF_3\right]^+\left[SbF_6\right]^-$

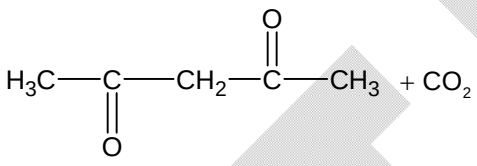
$P_4 + 8SOCl_2 \longrightarrow 4PCl_3 + 4SO_2 + 2S_2Cl_2$

27. C

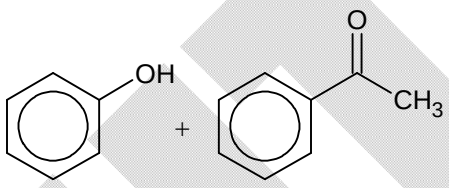
Sol. (P) $Mg(OH)_2 \downarrow + C_3H_4 \uparrow$

(Q) $C_2H_5 - CH_2 - CH_2 - NH_2 + HCOOH$

(R)



(S)



28. A

Sol. (P) Geometrical isomer – 5, enantiomer pair – 1, total stereo isomer – 6.

(Q) Geometrical isomer – 3, enantiomer pair – 1, total stereo isomer – 4.

(R) Square planar complex total 6 isomers (linkage, cis and trans)

(S) Total 6-stereo isomer are possible, 2 meso and 2 enantiomer pair.

SECTION – B

29. 10

Sol. $XeF_4 \rightarrow sp^3d^2$

$PH_3 \rightarrow P^3$ mixing

$\left[PdCl_4\right]^{2-} \rightarrow dsp^2$

$K\left[AgF_4\right] \rightarrow dsp^2$

$\left[Ni(CN)_4\right]^{2-} \rightarrow dsp^2$

$BrF_3 \rightarrow sp^3d$

$SF_4 \rightarrow sp^3d$

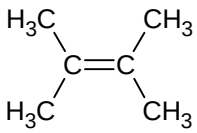
30. 11

Sol. I – 4, II – 2, III – 3, IV – 2

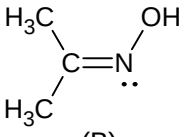
Total isomers = 11

31. 22

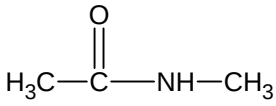
Sol.



(A)

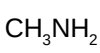


(B)



(C)

(D)



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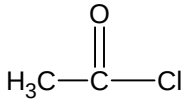

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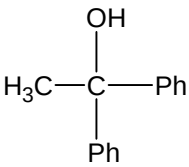
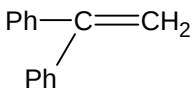
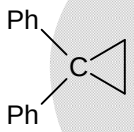


(G)



(H)



(I)

$x = 194 \;\; y = 9$

$$\Rightarrow \frac{x+4}{y} = \frac{194+4}{9} = \frac{198}{9} = 22$$

32. 4

Sol.

$$X(aq) \xrightarrow{t_{1/2}=15\ min} 2B(aq) + Y(aq)$$

$t = 0\ (mmole)$	x	0	0
$t = 15$ min	$x - \frac{x}{2}$	x	$\frac{x}{2}$
$t = 30$ min	$x - \frac{3x}{4}$	$\frac{3x}{2}$	$\frac{3x}{4}$
$t = \infty$	0	2x	x

Let the normality of (A) is N and initial volume of 'A' used 'a' ml.

at t = 0 m_{eq} of X = m_{eq} of A.

$x \times 2n = N \times a$ … (1)

Similarly applying law of equivalence at different time interval

$t = 15$ min $\quad \frac{x}{2} \times 2n + x \times n + \frac{x}{2} \times 2n = 40 \times N$ … (2)

at $t = 30$ min $\quad \frac{x}{4} \times 2n + \frac{3x}{2} \times n + \frac{3x}{4} \times 2n = N \times b$ … (3)

at $t = \infty$ $\quad 2x \times n + x \times 2n = N \times c$ … (4)

On solving all four equation

$$a = \frac{80}{3}ml, \quad b = \frac{140}{3}ml, \quad c = \frac{160}{3}ml$$

$$\frac{a+3c}{c} = \frac{80+480}{140} = \frac{56}{14} = 4$$

33. 19

Sol. $x = 7, y = 12$

$(p\pi - d\pi)$ bonds in SO_2, SO_3 and XeO_4 are 1, 2 and 4 respectively.

Banana bonds in B_2H_6, $Al(BH_4)_3$ and $Be(BH_4)_2$ is 2,6 and 4 respectively.

34. 12

Sol. $AB(s) \rightleftharpoons A(g) + B(g)$

$2-1 \qquad 1-x_1 \quad 1$

$A(g) \rightleftharpoons \frac{1}{2}C_2(g) + \frac{3}{2}D_2(g)$

$1-x_1 \qquad \frac{x_1}{2} \qquad \frac{3x_1}{2}$

$\frac{3x_1}{2} = 0.75$

$x_1 = \frac{1}{2}$

$a = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}(\text{mole/litre})^2$

$b = \frac{\left(\frac{3}{8}\right)^{3/2}\left(\frac{1}{8}\right)^{1/2}}{\left(\frac{1}{4}\right)} = \frac{3\sqrt{3}}{16}$

$x = [A] = \frac{1}{4}M \qquad y = [C_2] = \frac{1}{8}M$

$$\frac{b\sqrt{3}}{a(x+y)} = \frac{\frac{3\sqrt{3}\times\sqrt{3}}{16}}{\frac{1}{8}\left(\frac{1}{4}+\frac{1}{8}\right)} = \frac{\frac{9}{16}}{\frac{1}{8}\times\frac{3}{8}} = \frac{9}{16}\times\frac{64}{3} = 12$$

Mathematics

PART – III

SECTION – A
(One or More than one correct type)

*This section contains **THREE (03)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).*

35. B, C, D

Sol. R is mid-point of PQ $\Rightarrow$ P $\equiv$ (0, 0)

As equation of parabola is $(x-y)^2 + ax + by + c = 0$

$\Rightarrow x - y = 0$ is a line parallel to axis

$\Rightarrow$ Equation of axis is $(x-y) - (-12-0) = 0 \Rightarrow x - y + 12 = 0$

$\Rightarrow$ Equation of tangent at the vertex is $x + y + 6 = 0$

Hence, equation of parabola is $\left(\frac{x-y+12}{\sqrt{2}}\right)^2 = 4a\left(\frac{x+y+6}{\sqrt{2}}\right)$ and it passes through (0, 0)

$\Rightarrow a = 3\sqrt{2}$

36. A, B, D

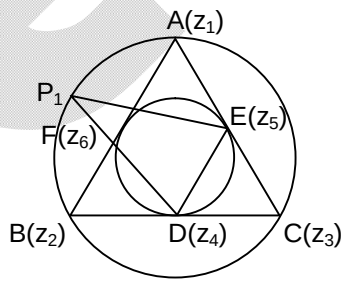
Sol. As ΔABC and ΔDEF are both equilateral

$\Rightarrow z_1 + z_2 + z_3 = 0$ and $z_4 + z_5 + z_6 = 0$

$\sum DP_1^2 = \sum |z - z_4|^2 = 3\left[|z_4|^2 + |z|^2\right] = 3[1+4] = 15$

Maximum area $A_1 = \frac{1}{2} DE \times \text{Height} = \frac{1}{2} \times \sqrt{3} \times \frac{5}{2} = \frac{5\sqrt{3}}{4}$

$\Rightarrow \frac{16}{5} A_1^2 = 15$



37. A, B, D

Sol. If $\alpha, \beta \in Q^C$:

Like rational then maximum two rational points on the circle

Unlike rational then maximum number of rational points is one

If $\alpha \in Q$ and $\beta \in Q^C \Rightarrow$ Maximum two rational points on the circle located in such a way that they are mirror image of each other about $x = \alpha$ line. For only one rational point, the point should be one end of vertical diameter of the circle

$\Rightarrow x_1 = 2$

38. C

Sol. Let tan x = t

Required limit = $\lim\limits_{t\to 0}\left[\left(\frac{1+t}{1-t}\right)^{\frac{1}{t}} - e^2\right]\frac{1}{t^2}dt$

$= \lim\limits_{t\to 0} \frac{e^{\frac{\ln(1+t)}{t}} - e^{2+\left(\frac{\ln(1-t)}{t}\right)}}{(1-t)^{\frac{1}{t}} \cdot t^2}$

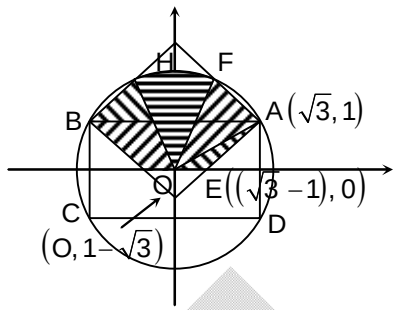
$= \lim\limits_{t\to 0} \frac{e\left[e^{-\frac{t}{2}+\frac{t^2}{3}-\frac{t^3}{4}+\ldots.} - e^{-\frac{t}{2}-\frac{t^2}{3}-\frac{t^3}{4}+\ldots.}\right]}{(1-t)^{\frac{1}{t}} \cdot t^2} = \frac{2}{3}e^2$

39. C

Sol. Required area = 2[Area(ΔOEA) + Ar(ΔOAF) + Ar(sector OFH)]

$= 2\left[\frac{1}{2}\times(\sqrt{3}-1)\times 1+\frac{1}{2}\times 2^2\times\sin 30^\circ+\frac{1}{2}\times 2^2\times\frac{\pi}{6}\right]$

$= \sqrt{3}-1+2+\frac{2\pi}{3}=\sqrt{3}+1+\frac{2\pi}{3}$



40. D

Sol. We have $S_1+S_2=\sum_{r=1}^{10}\sum_{q=1}^{10}\tan^{-1}\left(\frac{q}{r}\right)=\frac{1}{2}\sum_{r=1}^{10}\sum_{q=1}^{10}\left(\tan^{-1}\frac{q}{r}+\tan^{-1}\frac{r}{q}\right)=25\pi$

41. B

Sol. As $25-9=2+k+14-k \Rightarrow a_1^2-b_1^2=A_1^2+B_1^2$

Where a_1, b_1 represent the lengths of semi-major, semi-minor axes of ellipse and A_1, B_1 represent the lengths of semi-transverse and conjugate axes of hyperbola respectively

$\Rightarrow$ Ellipse and hyperbola are orthogonal

$\Rightarrow$ If equation of tangent to H at A is $ax-\sqrt{3}y-8=0$

$\Rightarrow$ Equation of tangent to H (i.e. normal to E) at D is $ax+\sqrt{3}y-8=0$

$\Rightarrow -ax-\sqrt{3}y+8=0 \Rightarrow a=5$ and $b=-\sqrt{3} \Rightarrow a+b=5-\sqrt{3}$

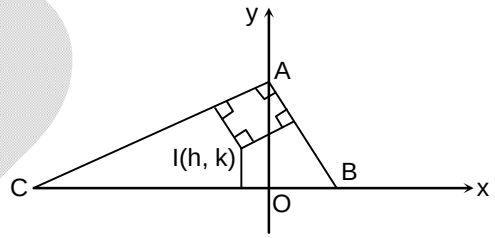
42. D

Sol. We have the lines $y[y^2-x^2-2y+1+\lambda(xy-x)]=0$

One line is y = 0 and other two variable lines are perpendicular and concurrent at (0, 1)

$\Rightarrow$ Orthocentre is at (0, 1) and clearly circumcentre lies on x-axis

We have AI = $\sqrt{2}k$



$\Rightarrow (h-0)^2+(k-1)^2=\left(\sqrt{2}k\right)^2 \Rightarrow x^2-y^2-2y+1=0$

Similarly locus of ex-centre is the same curve

43. B

Sol. (P) Let 1 be the image of first x_1 elements of set A and 2 be the image of next x_2 elements of set A and so on

$\Rightarrow x_1+x_2+x_3+x_4=6$; for onto function $x_i \geq 1$

$\Rightarrow$ Number of onto function = coefficient of x^6 in $(x+x^2+x^3+.....)^4$

= coefficient of x^2 in $(1-x)^{-4} = {}^{4+2-1}C_2=10$

$\Rightarrow$ Total number of onto function (non-increasing or non-decreasing) = 20

Again total number of into and onto function (non-increasing or non-decreasing)

$=\left(2\times {}^{4+6-1}C_6-4\right)=164$

(R) As $f(i)\neq i \Rightarrow$ each element has 3 choices i.e. total number of ways = $3^4=81$

But in these 3^4 cases D_4 cases are onto

$\Rightarrow$ Required number of ways = $81-D_4=81-9=72$

(S) Required number of function = $6\cdot5\cdot4\cdot3-\left\{{}^4C_1\cdot5\cdot4\cdot3-{}^4C_2\cdot4\cdot3-{}^4C_4\cdot1\right\}=181$

44. C

Sol. If B is taken is origin and BD as x-axis and plane BDC in xz plane

$\Rightarrow$ A $\equiv\left(\sqrt{2},2,2\right)$ and C $\equiv\left(4\sqrt{2},0,2\sqrt{2}\right)$

So, shortest distance between AB and DC is $\left|\frac{\begin{vmatrix} 5\sqrt{2} & 0 & 0 \\ \sqrt{2} & 2 & 2 \\ \sqrt{2} & 0 & -2\sqrt{2} \end{vmatrix}}{\begin{vmatrix} i & j & k \\ \sqrt{2} & 2 & 2 \\ \sqrt{2} & 0 & -2\sqrt{2} \end{vmatrix}}\right| = \frac{10}{\sqrt{4+\sqrt{2}}}$

45. A

Sol. Clearly ΔDEF will be Pedal triangle of ΔABC

As G is mirror image of a P (orthocentre) about side BC

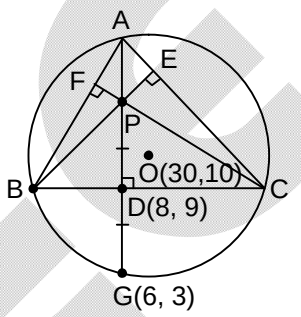
$\Rightarrow$ D is mid-point of PG $\Rightarrow$ P $\equiv$ (10, 15)

As PD = 2R cos B cos C

$\Rightarrow \sqrt{2^2+6^2} = 2\times\sqrt{(30-6)^2+(10-3)^2}\times\cos B\cos C$

$\Rightarrow 2\sqrt{10} = 2\times 25\times\cos B\cos C$

$\Rightarrow \sqrt{\frac{10}{k^2}} = 25$



Equation of circumcircle is $(x-30)^2 + (y-10)^2 = 25^2$ ..... (1)

and equation of side BC, $x + 3y - 35 = 0$ ..... (2)

and equation of line AD, $3x - y - 15 = 0$ ..... (3)

Solving (1), (2) and (3), we get A, B and C as (15, 30), (50, –5) and (5, 10)

$\Rightarrow$ AP $= 5\sqrt{10}$

$\Rightarrow$ Area of triangle ABC $= \Delta = \frac{1}{2}\times 15\sqrt{10}\times 7\sqrt{10} = 525$ sq. units

SECTION – B

46. 9

Sol. For $\left|\frac{a\alpha^7 + b\beta^4 + c\gamma}{a\beta + b\gamma^4 + c\alpha^7}\right| = 1$

$\Rightarrow$ When α, β, γ are different, then triplet (α, β, γ) can be arranged in 3! = 6 ways

If $\alpha = \beta = \gamma$, then 3 ways

$\Rightarrow$ Total number of ways = 6 + 3 = 9

47. 3

Sol. Required series is = $\tan^{-1}\left[\frac{2\sqrt{3}}{6}\right] + \tan^{-1}\left[\frac{2\sqrt{3}}{11}\right] + ...$

$= \tan^{-1}\left[\frac{\frac{2}{\sqrt{3}}}{1+\frac{3}{\sqrt{3}}\cdot\frac{1}{\sqrt{3}}}\right] + \tan^{-1}\left[\frac{\frac{2}{\sqrt{3}}}{1+\frac{4}{\sqrt{3}}\cdot\frac{2}{\sqrt{3}}}\right] + ...$

$= \left(\tan^{-1}\frac{3}{\sqrt{3}} - \tan^{-1}\frac{1}{\sqrt{3}}\right) + \left(\tan^{-1}\frac{4}{\sqrt{3}} - \tan^{-1}\frac{2}{\sqrt{3}}\right) + ... + \left[\tan^{-1}\left(\frac{n+2}{\sqrt{3}}\right) - \tan^{-1}\frac{n}{\sqrt{3}}\right]$

$S_\infty = \pi - \tan^{-1}\frac{1}{\sqrt{3}} - \tan^{-1}\frac{2}{\sqrt{3}} = \frac{5\pi}{6} - \tan^{-1}\frac{2}{\sqrt{3}} = \theta$

$\Rightarrow \cot\left(\frac{5\pi}{6} - \theta\right) = \cot\left[\frac{5\pi}{6} - \left(\frac{5\pi}{6} - \cot^{-1}\frac{\sqrt{3}}{2}\right)\right] = \frac{\sqrt{3}}{2}$

48. 11

Sol. We have, $S = \sum\sum \frac{1}{bc(b+c+2)} = \sum\sum \frac{1}{bc}\int_0^1 x^{b+c+1}dx$

$= \int_0^1 x\left(\sum\sum \frac{x^{b+c}}{bc}\right)dx = \int_0^1 x\left(\sum_{b=1}^{\infty}\frac{x^b}{b}\right)\left(\sum_{c=1}^{\infty}\frac{x^c}{c}\right)dx$

$= \int_0^1 x\left(\sum_{b=1}^{\infty}\frac{x^b}{b}\right)^2 dx = \int_0^1 x\ln(1-x)^2\,dx$

$= \int_0^1 (1-x)\ln(x)^2\,dx = \frac{7}{4} = \frac{p}{q} \Rightarrow p + q = 11$

49. 25

Sol. We are given $f(x) = \frac{2^{23}x + 2^{24}}{x^2 + 2^{24}x + 2^{24}}$

$\Rightarrow f\left(\frac{2^{24}}{x}\right) = \frac{2^{23}\left(\frac{2^{24}}{x}\right) + 2^{24}}{\left(\frac{2^{24}}{x}\right)^2 + 2^{24}\times\frac{2^{24}}{x} + 2^{24}} = \frac{2^{23}x + x^2}{2^{24} + 2^{24}x + x^2}$

[Multiplying numerator and denominator by $\frac{x^2}{2^{24}}$

Now on adding $f(x) + f\left(\frac{2^{24}}{x}\right) = 1$

$\Rightarrow$ Required sum = (f(1) + f(2^{24}) + (f(2) + f(2^{23})) + ... + (f(2^{11}) + f(2^{13})) + $\frac{1}{2}\left(f(2^{12}) + f(2^{12})\right)$

$= \frac{25}{2}$ $\therefore$ 2k = 25

50. 1

Sol. We have, $\begin{vmatrix} 1+2x^7 & 1+x^4 & 1-x^1 \\ 2-x^4 & 2+x^5 & 3+x^6 \\ x^1 & 1+x^2 & 1-x^3 \end{vmatrix} = -4\left[x^{15} + ()x^{14} + ... + (\ )x^0\right] =$

$-4(x-\alpha_1)(x-\alpha_2).....(x-\alpha_{15})$

Putting x = 0 on both sides $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 2 & 3 \\ 0 & 1 & 1 \end{vmatrix} = -4(-\alpha_1\cdot-\alpha_2\cdot.....\cdot-\alpha_{15}) = 4(\alpha_1\alpha_2\ .....\ \alpha_{15})$

$\Rightarrow -1 = 4(\alpha_1\alpha_2\ .....\ \alpha_{15}) \Rightarrow p = -\frac{1}{4} \Rightarrow 16p^2 = 1$

51. 103

Sol. $n(A) = 100^3 - 80^3$

$n(B \cap A) = (100^3 - 60^3) - (80^3 - 40^3)$

↓ Minimum at most 40 & maximum upto 100; ↓ Minimum [41, 80] Maximum upto 80

Required probability = $\frac{100^3 - 60^3 - 80^3 + 40^3}{100^3 - 80^3} = \frac{42}{61} = \frac{p}{q} \Rightarrow p + q = 103$